FORECASTING DEMAND USING SURVIVAL MODELING: AN APPLICATION TO US PRISONS

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ABSTRACT

A systems approach to modeling demand which incorporates survival modeling is applied to the problem of prison population projection. The approach models the flow of inmates through the prison system and differs from earlier approaches by exploiting the differences in the incarceration hazard rates of individuals in the general population and those who have previously been incarcerated and explicitly considering the impact of constrained prison capacity on release policy and future admissions. The methodology capitalizes on the impact of recursion in the prison population and reduces the amount and complexity of data required for long-term forecasts. First-time arrivals to prison are modeled as a Poisson process arising from the general population; recidivist arrivals are modeled using a failure model, where the reincarceration hazard rate is a function of age and race. The model is demonstrated for the state of North Carolina located in the Southeastern region of the United States. The effect of limited prison capacity on the mean of the time-served distribution is shown. The results demonstrate that an early release policy will generate an increase in prison admissions through the return to prison of former inmates. Further, the results show that a systems approach to modeling of prison demand which includes the non-linear effect of recidivism, i.e., survival modeling, has a significant impact on the accuracy of forecasts.

INTRODUCTION

Prison crowding is one of the most serious domestic issues currently being faced by the United States. Between 1980 and 1988, the number of inmates in state and federal prisons increased more than 90 percent, from 329,821 to 627,402 (US Department of Justice, Bureau of Justice Statistics, 1989). At the end of 1992, 43 state jurisdictions and the Federal prison system were operating at 100 percent or more of their prison capacities (US Department of Justice, Bureau of Statistics, 1993, p. 6). Recently, “get tough” sentencing policies—including “three strikes and you’re out” provisions have been enacted in several states. These policies imply that if an offender is found guilt of a violent crime three times, they will receive a mandatory sentence of life imprisonment. Such policies, although popular with a voters frustrated by the escalating nature of crime in the US, will only exacerbate the current crowding situation. For example, the state of California’s prison population was 115,534 on June 30, 1993 (US Department of Justice, Bureau of Justice Statistics, press release, October 3,1993) and the California Department of Corrections estimates that recently enacted “Three Strikes” legislation will increase incarceration by 81,628 prisoners by the turn of the century (California Department of Corrections, 1994).

The current crowding conditions in US prisons are due, in part, to a failure to predict the long-term demand for prison capacity. While adequate one-year-hence forecasts are possible with simple tools such as moving averages or other linear models, long-term forecasts are difficult to obtain. This is due, in part, to the nature of the prison population. Specifically, a recent survey showed that recidivists, those returning to prison, comprise about half of all prisoners in this country (Beck, et al., 1993, p.11). The effect of this subgroup on the flow of inmates through prison and on capacity is synergistic. That is, recidivists are more likely to receive longer sentences and thus increase demand for prison beds. The effect of insufficient capacity also has a synergistic effect with recidivists (see Lattimore and Baker, 1992, for a discussion of this point).

For example, if capacity is limited, then one of the few options open to prison administrators (assuming a lack of concern about prison conditions) is to release inmates earlier, hence recidivists are free sooner and "available" to commit a crime and be reincarcerated—the "revolving door" of the criminal justice system. Thus, the non-linearity in the long-term input-output process which characterizes the prison system has made traditional modeling techniques and traditional

* Points of view are those of the author and do not necessarily represent the official position of the US Department of Justice or the National Institute of Justice.
linear approaches inadequate for accurate forecasting of demand. As succinctly stated by MacDonald (1989), “Reality has had a habit of outstripping forecasts.”

Determining the future capacity needs for institutions in which demand is characterized by input and output processes that are stochastic in nature, driven, wholly or in part, by a population demographics, and subject to policy intervention is a difficult problem for planners. In addition to prisons, other systems which share some or all of these characteristics include hospitals and, to a lesser extent, school systems. Traditional approaches to predicting future demand for these types of systems include time series, econometric models, simulation studies and stochastic techniques (e.g., see Rhodes, 1990). Generally, these approaches require accurate estimates of the input population, disaggregated on the basis of historical data or a priori assumptions concerning changes in population demographics and trends. For example, a prison population at time $t$ may be represented as a mapping of the general population at time $t-1$ onto the prison population at time $t$. Thus, the demand-generating population is exogenous to the model, making predictions dependent on accurate characterization of the input process. These models ignore the endogenous effect of the process on the future intake population, and the effect of policy initiatives on future capacity.

We develop an approach to predicting demand for capacity which explicitly considers the feedback effect of the output process on the input process. We apply the methodology to prison population projection, extending some earlier work of Blumstein, Cohen and Miller (1980) and Barnett (1987). Our approach explicitly includes recidivists, incorporating the techniques of survival modeling into the stochastic modeling approaches of Blumstein, Cohen and Miller (BCM) and Barnett. The prison population projection model developed herein assumes that the prison intake population is generated from two distinct populations—the general population and the population of former inmates. Thus, intake is comprised of those coming to prison for the first time (“first-timers”) and those returning to prison (recidivists). Secondly, our model explicitly considers the impact of limited capacity on service rate, current prison population and future prison population. Although our model is not limited to prison application, indeed the generalization to demand for hospital beds, for example, is conceptually straightforward, the application allows us to call upon the theoretical work of Barnett and BCM and address a problem which is of increasing public concern, namely the problem of prison crowding. The state of North Carolina prison system will be used to demonstrate the model. The system selected has particular relevance to the modeling approach as in 1986 the state of North Carolina instituted a capacity ceiling in response to prison overcrowding and thus provides an opportunity for a model such as ours, which considers the impact of constrained capacity on the population forecast, to be validated.

The next section briefly examines the problem of prison overcrowding and reviews research in prison population project. Subsequent sections present the proposed projection model assumptions and formulation, and the data and estimation technique. Demonstration of the model for the state of North Carolina under assumptions of unconstrained and constrained capacity is presented.

**PRISON POPULATION PROJECTION**

Methods used to predict prison population include a variety of models, ranging from simple linear projections to microsimulation models (US General Accounting Office, 1984; see also Rhodes, 1990). One of the first probabilistic models for predicting inmate (and parolee) population was developed by Stollmack (1973) who derived a model from deterministic differential equations representative of the correctional system input and output (i.e., admissions and releases). The system of differential equations was shown to yield a model identical to an infinite-server queuing system with exponential service times. He developed the model for the case of a general service-time distribution (M/G/1/$\infty$) and projected estimates for the District of Columbia jails.

The model included a constant intake or commitment rate, $\gamma$, an exponential service distribution with service rate, $\mu$, and associated service time $S$. Prison population at time $t$, $P_t$, was estimated recursively, by the following summation of retention and intake:

$$P_t = P_{t-1} \cdot e^{-\mu} + \gamma \cdot S \cdot (1 - e^{-\mu}),$$

where the first term on the right-hand side is the number of prisoners at time $t-1$ who remain in prison at time $t$ and the second term is the number of individuals committed to prison in the period $(t-1, t)$ who remain in prison at time $t$. BCM (1980) extended the approach of Stollmack by disaggregating the inputs by age ($a$), sex ($s$), race ($r$), and offense ($o$) type. Information on criminal justice system processing rates (arrest, indictment, conviction, and incarceration) was used to develop demographic-and-offense-specific arrival rates ($\gamma_{aro}$). The service rate, $\mu_{aro}$, associated with the age-race-sex-offense-specific mean time served $S_{aro}$ were also estimated. This information was then used to estimate equation (1) for each $aro$ group. Summing over these yielded the prison population at time $t$; prison population forecasts for the State of Pennsylvania for the years 1970 through 2000 were made.

The BCM work was subsequently extended by Barnett (1987), who allowed more flexibility toward the modeling of policy changes. Arrivals in Barnett's model were from the “chronic offender population,” a subset of the general population assumed to begin crime at age $c$ and retire at age $r$. By modeling the processes governing retirement from...
crime and imprisonment, Barnett developed a status-quo model (i.e., assuming no changes in incarceration policy) of the following form:

\[ P_t = \int_c p(a) \cdot PR(a) \, da \]  

(2)

where \( p(a) \) is the age-specific density of chronic offenders and \( PR(a) \) is the probability that offenders of age \( a \) will be incarcerated at time \( t \). Barnett assumed one homogenous, chronic-offender class characterized by the same starting age, retirement and incarceration processes, and sentencing distribution. Chronic offenders were assumed to be a known proportion of the general population and the proportion of offenders of various ages, races, etc., were assumed to remain constant over time. Barnett used data from the BCM study to arrive at his own projections state of Pennsylvania prison population. He also made status-quo projections for other states in the US, including Massachusetts, Florida, and Utah, and projections which assumed that policy changes would result in increased (mean) time served. Finally, Barnett used his model to examine the impact of changes in the assumed proportion of individuals in the chronic offender population, and sentencing policy.

The forecasting approaches of BCM and Barnett take advantage of information that can be known with some confidence (i.e., general population forecasts) to improve prison population projections over simpler methods which, for example, project next year's prison count on the number of individuals in prison this year. Two shortcomings of their approaches are 1) the impact of recidivism on the intake population cannot be addressed endogenously, and 2) the impact of capacity on time served and therefore its effect on the return of recidivists cannot be ascertained. By separately modeling the return to prison of those previously incarcerated (about 50 percent of all prison admissions) two benefits are derived. Firstly, the non-constant (with respect to time) hazard rate of repeat offenders can be used to more accurately model the "career criminals" discussed by Barnett (1987). Secondly, the impact of capacity constraints and changes in sentencing policy on future admissions can be evaluated.

**MODEL**

A recidivism-inclusive-population-projection model is developed as an input-output model. Inputs are assumed to be stochastic in nature, following a well-defined probability distribution. Similarly, outputs are generated as a function of a well-defined distribution over time served. The "stocks" at any time \( t \), include the general population \( N_t \), the prison population \( P_t \), and the population of released offenders \( R_t \). "Flows" through the system in the interval \( (t - 1, t] \) include first-timer commitments from the general population \( C_{t-1,t}^F \), recidivist commitments from the former inmate population \( C_{t-1,t}^R \), and releases \( R_{t-1,t} \). This process is illustrated in Figure 1.

The \( \phi_t^F \) and \( \phi_t^R \) are probability density functions that describe the probability that an individual will be incarcerated at time \( t \) for the first time from the general population, and reincarcerated from the released population, respectively. These distributions represent the joint probability of arrest, indictment, conviction, and incarceration for each group. For the moment, we assume homogeneity of the two arrival-generating populations, \( N_t \) and \( R_t \). Thus, the components of the model can be defined as follows:

\[
\begin{align*}
N_t &= \text{the number of individuals in the general population at time } t; \\
R_t &= \text{the number of former inmates free at time } t; \\
P_t^F &= \text{the total number of inmates at time } t \text{ who are incarcerated for the first time}; \\
P_t^R &= \text{the total number of inmates at time } t \text{ who have at least one prior incarceration}; \\
P_t &= \text{the total prison population at time } t = P_t^F + P_t^R; \\
C_{t-1,t}^F &= \text{number of first-timer commitments in the period } (t - 1, t]; \\
C_{t-1,t}^R &= \text{number of recidivist commitments in the period } (t - 1, t]; \\
\phi_t^F &= \text{the probability of arriving to prison from the general population}; \\
\phi_t^R(i) &= \text{the probability of arriving to prison from the released population } R_i \text{ for } i < t; \\
\tau_t^F &= \text{the service rate for the first-timer prison population } (t - 1, t]; \text{ and} \\
\tau_t^R &= \text{the service rate for the recidivist prison population in period } (t - 1, t].
\end{align*}
\]

First-time commitments are assumed to arrive from the general population, \( N \), via a Poisson distribution with constant mean \( \phi_t^F \). Total commitments from this population in time \( (t - 1, t] \) are:
Figure 1. Model Stocks and Flows

- General Population, \( N \)
- Released Population, \( R \)
- Prison Population, \( P \)

Flow arrows:
- \( C^F \) from \( N \) to \( P \)
- \( C^R \) from \( P \) to \( R \)
- \( R \) to \( P \)
Commitments from the released population, R, arrive via a split-lognormal failure model. The lognormal failure function was chosen because the hazard rate of this distribution, which increases and then decreases, is consistent with observed recidivism patterns (see, for example, Schmidt and Witte, 1988). As the hazard rate associated with the lognormal distribution is not constant, the probability of returning to prison at time \( t \) is conditional on the time since release. Specifically, the probability of returning to prison in period \((t - 1, t]\), given release in period \((i-1, i]\), \( i < t \), is:

\[
\phi^R_i (i) = \delta \cdot \int_{t-i}^{t-1} f(x)dx
\]

where

\[
f(x) = \exp\left[-\ln(x - \mu)^2 \cdot (2\sigma^2)^{-1}\right] / \left(x\sigma \cdot \sqrt{2\pi}\right)
\]

and \( \delta, \mu, \) and \( \sigma \) are maximum likelihood estimates of the split-lognormal survival model. The splitting parameter, \( \delta \), "splits" the released population into two groups, those who will eventually recidivate and those who will not. Inclusion of this parameter adapts the more familiar failure model to the situation in which not all individuals ultimately fail and allows explicit consideration of the career criminal or chronic offender paradigm since desistance (or "retirement") is accommodated by this parameter. Thus, commitments in period \((t - 1, t]\) from the released population are:

\[
C_{t-1,t}^R = \sum_{i=1}^{t-1} \phi^R_i (i) \cdot R_{i-1,i}
\]

As can be seen, the number of commitments from the released population is a function not only of the total number of released individuals at risk in the period \((t - 1, t]\), but also of the time since their releases. Reizations from prison are assumed to follow a negative exponential distribution, where sentence length (service time) is dependent upon whether the inmate is a first-timer or a recidivist. These distributions have means \( \tau_i^F = S_i^F \), for first-timers, and \( \tau_i^R = S_i^R \) for recidivists, where \( S_i^F \) and \( S_i^R \) are estimates of the mean time served for first-timers and recidivists, respectively.

The general model for predicting prison population at time \( t \) is thus:

\[
P_t = P_{t-1}^F \cdot e^{-\tau_t^F} + P_{t-1}^R \cdot e^{-\tau_t^R} + C_{t-1,t}^F \cdot S_t^F \cdot (1-e^{-\tau_t^F}) + C_{t-1,t}^R \cdot S_t^R \cdot (1-e^{-\tau_t^R})
\]

The number of individuals in prison at time \( t \) is the sum of those admitted prior to time \( t \) who have not been released by \( t \), plus those admitted and not released between \( t - 1 \) and \( t \). Iteration of this model produces subsequent population projections. It should be noted that the prison admission rate is not necessarily constant over time, but instead is a function of the number of released individuals and the timing of their release. Note that a constant overall admission rate implies \( \phi_{t+1} = \phi_t \), or that

\[
\phi^{N} \cdot N_{t+1} + \sum_{i=1}^{t} \phi^R_{t+1} (i) \cdot R_{i-1,i} / N_{t+1} = \phi^{N} \cdot N_t + \sum_{i=1}^{t-1} \phi^R_{t+1} (i) \cdot R_{i-1,i} / N_t
\]

There is no reason to assume that this equality will hold, particularly in times of large demographic changes in the populations.

The model presented in equation (7) assumes only two homogeneous populations from which prison admissions are generated. As propensity to crime varies by age, race and sex, improvements in projections can be achieved by estimating the model for specific demographically homogeneous classes, as suggested by Stollmack (1973) and incorporated into the models of BCM (1980) and Barnett (1987). Thus, equation (7) will be estimated for age/race/sex specific classes and the total \( P_t \) will be found by summing over the classes.

When different age groups are considered, a small complication arises with respect to "aging" the recidivist commitments. Specifically, the probability of returning to prison is a function of age at release from the last admission to prison. By equation (6), the number of recidivist commitments of age class \( A \) in period \((t - 1, t]\) is:

\[
C_{t-1,t}^{A,R} = \phi^A \cdot N_t
\]
\begin{equation}
C_{t-1,t}^R (A) = \sum_{i=1}^{t-1} \phi_i^R (i, A) \cdot R_{t-1,i}^R (A).
\end{equation}

where $\phi_i^R (i, A)$ is the probability that a member of age class A at the previous admission returns to prison in period (i-1, i] given release in period (i-1, i] and $R_{t-1,i}^R (A)$ is the number of releases who were in age class A when admitted prior to release in period (i-1, i]. However, some of the $R_{t-1,i}^R (A)$ individuals will have aged into an older class prior to this new commitment and some members of younger classes will have aged into class A. Thus, aging of the commitment population is necessary.

Finally, the model presented in equation (7) assumes that $P_t < P_t^{Cap}$ for all $t$. Rather than population, this model actually predicts demand or what Stollmack (1973) referred to as "population pressure." Imposition of a capacity constraint, e.g. $P_t^{Cap}$, can be operationalized by assuming that the $S$ will be reduced as prisoners are released early to meet the capacity constraint. Under a status-quo assumption (meaning that prisoners incarcerated at time $t=0$ will serve the same sentence as those imprisoned in $t=t+n$) and given that $P_t$ and $C_{t-1,t}$ are known, the number of releases (from each prison population) required in the interval (t - 1, t] is:

\begin{equation}
R_{t-1,t} = P_{t-1} \cdot (1 - e^{-\tau}) + C_{t-1,t} \cdot (1 - \tau^{-1} \cdot (1 - e^{-\tau}))
\end{equation}

The number of releases required, however, is

\begin{equation}
R_{t-1,t}^{Cap} = R_{t-1,t} + P_t - P_t^{Cap}
\end{equation}

Given that $R_{t-1,t}^{Cap}$ is known, the following equation identifies $\tau^*$ and the time served distribution required to satisfy the capacity constraint:

\begin{equation}
R_{t-1,t}^{Cap} = P_{t-1} \cdot (1 - e^{-\tau^*}) + C_{t-1,t} \cdot (1 - \tau^{-1} \cdot (1 - e^{-\tau^*}))
\end{equation}

A search routine (bisection method) was used to identify $\tau^*$. When $P_t^{Cap} > P_t$, status-quo prison population projections are obtained from equation (7), thus providing an upper bound on expected demand. When $P_t^{Cap} \leq P_t$ and $P_t^{Cap}$ is known, the new service times needed to meet available capacity can be obtained.

**DATA AND ESTIMATION METHODS**

Data were obtained for the state of North Carolina. Population estimates by age and race groups were provided by the North Carolina Office of Budget and Management for 1980 through the year 2000. The North Carolina Department of Correction provided information on the probability of first-time incarceration by age and race group, average annual prison population (1979 through 1988), and the number of releasees in 1979. Data to estimate the recidivism models were obtained from the Inter-University Consortium of Political and Social Research. These data comprise the 1980 North Carolina release cohort data set described in Schmidt and Witte (1988). This data set (hereafter referred to as the S&W sample) contains recidivism information for 9,549 prisoners released from North Carolina prisons between July 1, 1979, and June 30, 1980.

Projections will be made for "classes" of individuals. As male prisoners comprise about 96 percent of the prison population in North Carolina, forecasts will be made only for males. Specifically, first-timer and recidivist commitments and prison populations will be estimated by race (white and non-white) and age. Seven age categories will be used: [15,20), [20, 25), ..., [40,45), and [45+) years. Thus, we have 14 classes for each of our two populations.

The model was initialized using data for the year 1979 ($t=0$) and estimates of commitments, releases, and prison populations were generated for the years 1980 through 2000. The average 1979 male prison population ($P_0$) was 13,489. To properly initialize the model, the population was disaggregated by age, race and previous incarceration status (first-timer-versus-recidivist) as shown in Table 1.

The number of males released from prison in 1979 was estimated to be 8,919. This total was disaggregated into age/race categories using the age/race distribution of the S&W sample. This distribution is also included in Table 1. As this release cohort represents only a fraction of the total number of previously incarcerated individuals "on the street" in North Carolina in 1980, it was necessary to estimate a larger, more accurate $R_0$. The number of male prisoners released for 1959 through 1978 were developed using data from volumes of the Sourcebook of Criminal Justice Statistics (US Department of Justice, 1975, 1976, 1977, 1978, 1979, 1980). The number who were still on the street at $t = 0$ were estimated as $R_{t-1,t}$ times one minus the cdf (at $t = 21, 20, ..., 1$ years) of the estimated split-lognormal failure model (see

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Table 1: Initial Prison and Releasee Age/Race Distribution (%)

<table>
<thead>
<tr>
<th>Age Group</th>
<th>First-Timers</th>
<th></th>
<th></th>
<th>Recidivists</th>
<th></th>
<th></th>
<th>Releases</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>White</td>
<td>Non-White</td>
<td>White</td>
<td>Non-White</td>
<td>White</td>
<td>Non-White</td>
<td>White</td>
<td>Non-White</td>
<td>White</td>
</tr>
<tr>
<td>15-20</td>
<td>9.38</td>
<td>7.38</td>
<td>2.12</td>
<td>1.77</td>
<td>14.04</td>
<td>11.65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20-25</td>
<td>9.38</td>
<td>8.15</td>
<td>5.20</td>
<td>5.32</td>
<td>13.70</td>
<td>13.49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25-30</td>
<td>3.84</td>
<td>4.80</td>
<td>3.87</td>
<td>5.45</td>
<td>7.20</td>
<td>9.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-35</td>
<td>2.32</td>
<td>2.33</td>
<td>3.46</td>
<td>3.96</td>
<td>5.35</td>
<td>5.36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35-40</td>
<td>1.37</td>
<td>1.04</td>
<td>2.63</td>
<td>2.41</td>
<td>3.66</td>
<td>3.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40-45</td>
<td>0.92</td>
<td>0.8</td>
<td>1.92</td>
<td>1.48</td>
<td>2.84</td>
<td>2.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45+</td>
<td>1.50</td>
<td>0.95</td>
<td>3.40</td>
<td>2.81</td>
<td>4.5</td>
<td>3.44</td>
<td></td>
<td></td>
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<tr>
<td>Total</td>
<td>28.71</td>
<td>25.48</td>
<td>22.60</td>
<td>23.20</td>
<td>51.31</td>
<td>48.69</td>
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</table>

Lattimore and Baker, 1992). This iterative process generated an \( R_0 \) of 114,012.

Recall that commitments in each period \((t - t, t]\) derive from two populations, the general population, \( N_t \), and the releasee population, \( R_t \). Commitments from \( N \) were assumed to be generated by the distribution \( \phi^F \), the probability distribution of first-time incarceration. \( \phi^F (a, r) \) was calculated from information provided by the North Carolina Department of Corrections (see Table 2). As can be seen, the probability of a first-time incarceration increases from the youngest to the next youngest class and then decreases with age. This distribution over age is due to two factors. Firstly, younger adult males are more likely to commit crimes than older males; secondly, those admitted to prison at older ages are more likely to be recidivists. Commitments from the general population, \( C_{t, t+1}^F \), were obtained by multiplying \( \phi^F (a, r) \) by estimates of the general population by age/race for the years 1980 through 2000.

To estimate recidivist commitments, it was necessary to develop parameter estimates for the split-lognormal recidivism models (see equations 5-7). The S&W data were used to generate maximum-likelihood parameter estimates for each of the fourteen age/race groups. The actual and estimated numbers of failures for 15-to-20 year old white males are shown in Figure 2. The results for the other classes are similar suggesting that the models provide good fit to the data. The parameter estimates are shown in Table 3. The estimates imply a range of mean time to failure from 22.2 months for the youngest white group to 124.5 months for the (35,40) non-white group.

Table 2: Probability of First-Time Incarceration by Age and Race, \( \phi^F (a, r) \)

<table>
<thead>
<tr>
<th>Age Group</th>
<th>White</th>
<th>Non-White</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-20 years</td>
<td>0.0039</td>
<td>0.0129</td>
</tr>
<tr>
<td>20-25 years</td>
<td>0.0040</td>
<td>0.0143</td>
</tr>
<tr>
<td>25-30 years</td>
<td>0.0028</td>
<td>0.0141</td>
</tr>
<tr>
<td>30-35 years</td>
<td>0.0016</td>
<td>0.0107</td>
</tr>
<tr>
<td>35-40 years</td>
<td>0.0014</td>
<td>0.0058</td>
</tr>
<tr>
<td>40-45 years</td>
<td>0.0010</td>
<td>0.0040</td>
</tr>
<tr>
<td>45+ years</td>
<td>0.0003</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

As previously noted, prison releases are assumed to follow a negative exponential distribution. The mean of this distribution is the inverse of the mean time served, \( S \). These data were acquired from the 1980 release cohort data set. \( S \) was found to be significantly different for each age group; further, recidivists served longer sentences than first timers. Race was not a factor in the average time served. \( S \) by age and first-timer versus recidivist status are shown in Table 4.
Figure 2. Observed and Expected Reincarcerations

15-19-Year-Old White Males

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Table 3: Failure Model Survival Estimates

<table>
<thead>
<tr>
<th>Race</th>
<th>Age</th>
<th>Number</th>
<th>Failures</th>
<th>δ</th>
<th>μ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>(15, 20)</td>
<td>1204</td>
<td>529</td>
<td>0.481</td>
<td>2.68</td>
<td>0.927</td>
</tr>
<tr>
<td></td>
<td>(20, 25)</td>
<td>1195</td>
<td>370</td>
<td>0.390</td>
<td>3.04</td>
<td>1.090</td>
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<tr>
<td></td>
<td>(25, 30)</td>
<td>628</td>
<td>197</td>
<td>0.480</td>
<td>3.42</td>
<td>1.318</td>
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<tr>
<td></td>
<td>(30, 35)</td>
<td>481</td>
<td>151</td>
<td>0.462</td>
<td>3.25</td>
<td>1.479</td>
</tr>
<tr>
<td></td>
<td>(35, 40)</td>
<td>331</td>
<td>100</td>
<td>0.356</td>
<td>2.84</td>
<td>1.072</td>
</tr>
<tr>
<td></td>
<td>(40, 45)</td>
<td>246</td>
<td>84</td>
<td>0.557</td>
<td>3.47</td>
<td>1.643</td>
</tr>
<tr>
<td></td>
<td>(45+)</td>
<td>427</td>
<td>105</td>
<td>0.289</td>
<td>2.55</td>
<td>1.338</td>
</tr>
<tr>
<td>Non-White</td>
<td>(15, 20)</td>
<td>999</td>
<td>525</td>
<td>0.631</td>
<td>2.88</td>
<td>1.095</td>
</tr>
<tr>
<td></td>
<td>(20, 25)</td>
<td>1203</td>
<td>525</td>
<td>0.551</td>
<td>3.03</td>
<td>1.114</td>
</tr>
<tr>
<td></td>
<td>(25, 30)</td>
<td>839</td>
<td>327</td>
<td>0.478</td>
<td>2.96</td>
<td>1.093</td>
</tr>
<tr>
<td></td>
<td>(30, 35)</td>
<td>494</td>
<td>157</td>
<td>0.533</td>
<td>3.56</td>
<td>1.523</td>
</tr>
<tr>
<td></td>
<td>(35, 40)</td>
<td>309</td>
<td>99</td>
<td>0.564</td>
<td>3.68</td>
<td>1.513</td>
</tr>
<tr>
<td></td>
<td>(40, 45)</td>
<td>190</td>
<td>65</td>
<td>0.411</td>
<td>2.88</td>
<td>1.083</td>
</tr>
<tr>
<td></td>
<td>(45+)</td>
<td>320</td>
<td>92</td>
<td>0.335</td>
<td>2.71</td>
<td>1.136</td>
</tr>
</tbody>
</table>

Note: Values are maximum-likelihood parameter estimates for split-lognormal failure models estimated using 1980 North Carolina data.

Table 4: Mean Time Served by Age and Status (Years)

<table>
<thead>
<tr>
<th>Age</th>
<th>First-Timers</th>
<th>Recidivists</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-to-20 years</td>
<td>1.5</td>
<td>2.6</td>
</tr>
<tr>
<td>20-to-25 years</td>
<td>1.4</td>
<td>2.4</td>
</tr>
<tr>
<td>25-to-30 years</td>
<td>1.2</td>
<td>2.0</td>
</tr>
<tr>
<td>30-to-35 years</td>
<td>1.1</td>
<td>1.8</td>
</tr>
<tr>
<td>35-to-40 years</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>40-to-45 years</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td>45+ years</td>
<td>1.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

RESULTS FOR NORTH CAROLINA

We initially estimate North Carolina prison populations for the years 1980 through 2000 under the following status-quo assumptions:

1) The probability of a first-time incarceration (by age and race), $p_i^F$, is constant over the forecast period.
2) The probability distribution of a return to prison (by age and race) is constant over the forecast period and over the period in which the initial released population was generated.
3) $S$ (by age and prior-incarceration status) is constant over the forecast horizon.
4) The distribution of intra-class age-at-admission, in other words the proportion of each class of each age, is constant over the forecast horizon.
5) $p_i^{Cap} < p_i^F$, for all $t$.

These assumptions imply that there will be no changes in criminal justice system efficiency or policy nor changes in the propensity of individuals to commit crime and enter prison. The next subsection presents results under the assumptions presented above; subsequently, results are presented when assumptions 1, 3 and 5 are relaxed.

Status Quo Projections

Figure 3 shows the projected admissions and prison populations for the years 1980 through 2000. As can be seen, both prison admissions and prison population are projected to increase until about 1995 after which they stabilize. The male prison population is projected to increase by about 39 percent, from 15,177 in 1980 to 21,117 in the year 2000. Admissions are projected to increase from 10,400 to 13,800. For the forecast period, the general population in North Carolina is also projected to grow, but the general population growth is less than that of the projected prison population growth.
Status Quo Unconstrained Projections and Observed Values

Figure 3. Prison Populations and Commitments:

- P-obs
- C-Proj
- P-Proj

Year


Number of Incarcerations

2000 1600 1400 1200 1000 800 600

September 1994
The growth in admissions and prison population is largely attributable to increasing numbers of recidivists entering prison (and staying for longer periods). First-time admissions remain between about 7,000 and 7,500 throughout the forecast period. Recidivist admissions increase, however, from about 3,500 in 1980 to more than 6,200 in the year 2000. The increase in recidivist admissions is, of course, attributable to increases in releases over the planning horizon. As the prison population grows, the number of releases also grows, generating a larger population of potential recidivists. Indeed, over the 21-year forecast horizon, annual releases increase 43 percent (from 9,613 in 1980 to 13,741 in the year 2000).

As was noted in the previous section, the admission rate by age/race group may change over the forecast horizon. The data show that there is considerable growth in the admission rate for the older age groups over the forecast period, suggesting an "aging" of the prison population over time. Table 5 shows the projected changes in admission rates (per 100,000 population) between 1980 and 2000. The admission rates for the youngest (15-20 years) groups are relatively stable primarily due to the high proportion of first-time admissions in this class (88 percent in 1980; 85 percent in 2000) and the assumption that \( \phi^F(a,r) \) is constant over \( r \). The prison admission rates for older groups increase substantially over time—for example 22 percent for the 40-to-45-year-old white group and 47 percent for the 35-to-40-year-old nonwhite group.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Percentage Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>White</td>
</tr>
<tr>
<td>15-to-20 years</td>
<td>-0.02</td>
</tr>
<tr>
<td>20-to-25 years</td>
<td>-0.03</td>
</tr>
<tr>
<td>25-to-30 years</td>
<td>0.05</td>
</tr>
<tr>
<td>30-to-35 years</td>
<td>0.18</td>
</tr>
<tr>
<td>35-to-40 years</td>
<td>0.22</td>
</tr>
<tr>
<td>40-to-45 years</td>
<td>0.15</td>
</tr>
<tr>
<td>45+ years</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The estimations in this section assume that capacity is unlimited and \( \tau^F \) and \( \tau^B \) remain the same as that served by the 1980 release cohort. In the next section, we modify our model to allow for the impact of constrained capacity on prison population, mean time served, releases, and admissions. Subsequently, we consider the effects of potential changes in sentencing policy in a constrained and unconstrained environment.

**Capacity Constrained Projections**

After a brief turndown in 1982 through 1984, North Carolina prison population grew rapidly until 1986 after which it held steady at just under 17,000. There is a simple explanation for the relatively constant number of North Carolina prisoners over the last three years. In 1986, the North Carolina State Legislature imposed a three-year capacity constraint of 18,000 (male and female) inmates on North Carolina's prison population. The North Carolina Department of Correction administers this constraint by the early release of inmates when the prison population reaches 17,460. About 94.3 percent of North Carolina's prison capacity is available for male inmates implying a maximum allowable male prison population of 16,465. This legislated constraint of course mirrored available capacity.

If the prison population at time \( t \) exceeds the capacity, \( P_t^{\text{Cap}} \), additional releases are required. These "early releases" reduce \( S \) for the aggregate population. We initially assume that these additional releases are distributed proportionally across the 28 age/race/prior-incarceration classes. We then examine how the capacity ceiling of 16,465 affects mean time served and, subsequently, the distributions of returns to prison.

The effect of the capacity ceiling on mean time served was modeled. Results are shown in Figures 4 and 5. Figure 4 shows, for the youngest age groups, the change in the time served means required to achieve the capacity constraint. These and the results for the other groups suggest--other things equal--that a "steady state" is achieved fairly quickly under the constrained status quo conditions. The status quo and "steady state" means are shown in Figure 5. These results suggest that a decrease of about 25 percent in the means of the time-served distributions would be required. Although strictly comparable figures are not available from North Carolina, between 1983 and 1989, the average time served by released felons dropped by 12.8 percent (from 1.95 years to 1.7 years), while the average sentence length of those admitted has increased by 15.3 percent (from 6.0 to 7.2 years). The average time served by misdemeanants dropped by more than 40 percent during the same period.
A decreasing mean service time has the effect of increasing the releasee pool $R$, thereby increasing the number of admissions over the status quo model (assuming criminal behavior, as measured by the parameters of the survival model, does not change). The model predicts an additional 3,530 commitments due to the early release policy engendered by the capacity constraint. As the model is structured, these additional commitments are due entirely to the younger ages of release of the offenders. A second factor which could affect the rate of recidivist returns is a reduction in the deterrent effect that could result from reduced punishment.

CONCLUSIONS

A model which accurately predicts future demand for prison beds is important. However, most models which have attempted this task have offered projections which ignore the fact that there is not enough space to house inmates. Thus, these models are not only unrealistic, but limited in that they fail to take into account the synergistic effect of early release policies (instituted in response to overcrowding) and the intake process. Recall that close to fifty percent of the inmate population are recidivists. And a large percentage of these individuals are in crime-prone ages. Early release policies therefore tend to place highly active individuals on the street earlier allowing them more time to recidivate. In other words, the "revolving door" spins faster.

Underlying any discussion of capacity needs is the issue of what constitutes "optimal" capacity. Fiscal and humanitarian issues notwithstanding, the question of how many beds are needed must consider the impact of current and future capacity on demand. Population projection models which view the system as being incapacitated provide an upper bound of demand. Practically speaking, no jurisdiction is likely to build enough prisons to house all projected inmates. Because capacity will never meet demand, the problem becomes one of determining what level of capacity is acceptable. Results here suggest that this question cannot be answered completely unless the effect of reduced capacity on future demands is addressed. Also, if there is a deterrent effect and it is inversely related to time served (ceteris paribus), one could also see changes in the recidivism model. For example, if serving 18 months is more of a deterrent to future criminality than serving six months, then capacity constraints and resulting early release policies may have an even more dramatic effect on future admissions.

In this paper, we developed a stochastic model for an input/output process which exploits the effect of outputs on the input process. The model is the first to model the prison system from the vantage point of interrelated system in which the outputs reenter as inputs. This approach is a dramatic change from linear forecast models which view inputs and outputs to be linear in nature, to one in which the control of the system requires modeling of the non-linear properties of the population flow. Results reported suggest that a prison population projection model which includes the effect of recidivism on future demand provides a good forecast and, more importantly, allows for consideration of the effect of policy changes on the input/output process.
Youngest Age Group.

Figure 4. Impact of Capacity Constraint on Mean Time Served For

FT (15 - 20) = Rec (15 - 20)

Year


S (yrs)

0.5 1 1.5 2 2.5 3
Figure 5. Impact of Capacity Constraint on Time Served.

Age Group

FT-Initial FT-New Rec-Initial Rec-New

O 15- 20- 25- 30- 35- 40- 45- 45+ 0.5

5 (yrs) 3 2 1 0.5
REFERENCES