A mathematical model for the municipal solid waste location-routing problem with intermediate transfer stations

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Abstract

Municipal solid waste management is one of the challenging issues in mega cities due to various interrelated factors such as operational costs and environmental concerns. Cost, as one of the most significant constraints of municipal solid waste management can be effectively economized by efficient planning approaches. Considering diverse waste types in an integrated municipal solid waste system, a mathematical model of the location-routing problem is formulated and solved in this study in order to minimize the total cost of transportation and facility establishment. The main aim of the proposed model is to optimize the locations of the waste management system's facilities including transfer stations, treatment centres, recycling centres and disposal centres, and identify the optimal routes to and from the facilities. A case study of the location-routing problem in New South Wales, Australia, is investigated and analysed. Twenty five districts are selected to represent the whole state of New South Wales for the analysis. Twenty-four districts are also assumed to be candidate locations for each of the six types of the system's facilities which leads to a network with 24*6 potential location nodes i.e. 24^6 possible solutions. The presented application in this study is by far the largest size of the network among the related studies in the literature.

Keywords: Location-routing problem; Municipal solid waste; Hazardous waste; Routing; Facility location

1 Introduction

Business analytics approaches have received growing attention in recent years as to how various computer technologies can be utilized to provide decision makers with ad hoc analysis capabilities. New business models based on analysis of cost and quality have been adopted in operational and strategic decision-making. Apart from cost and quality, business models have also taken account of environmental issues such as emission and energy consumption of production facilities and pollution from logistics (Dawson and Spedding, 2009). The reduction of waste management costs and the increase of the benefits from recycling have been perceived as a new competitive business for many large-scale private sectors and government organizations. However, investigations on how business analytics can be applied to waste management are rare in the literature.

Municipal Solid Waste Management (MSWM) is considered as one of the most challenging issues in many populous cities. Generally, MSWM includes the processes associated with collection, transportation, treatment, recycling and disposal of waste in a safe, hygienic and cost-effective manner. A successful MSWM requires the appropriate site selection of the waste management system's facilities such as recycling and disposal facilities, and transportation of wastes among the facilities. The extensiveness and complexity of the factors affecting MSWM (e.g. limited resources such as land, investment costs and operational costs) make it difficult to be properly implemented.

There are different kinds of municipal solid waste (MSW) such as rubbish, food waste, commercial waste, industrial waste, construction waste and sanitation waste. Generally, MSW includes recyclables (such as paper, glass and plastics), toxic substances (paint, pesticides, used batteries, medicines), compostable organic matter (fruit and vegetable peels, food waste) and soiled waste (blood stained cotton, disposable syringes, etc.) (Sharholy et al., 2008). However, in a broader classification perspective we can divide MSW into the three main categories: hazardous wastes, recyclable wastes and garbage.

Hazardous wastes are harmful to human health and animals, and can have destructive environmental impacts if they remain in the environment and residential areas. Hazardous wastes can be classed as four main types: (i) ignitable wastes, (ii) wastes with the property of corrosiveness, (iii) wastes with the property of chemical reactivity, and (iv) toxic wastes. Hazardous wastes must be treated under certain technologies such as incineration and chemical treatment, and the material which is recyclable after treatment can be recycled. The rest must be disposed in residues disposal sites after removing the hazardous properties (Samanlioglu, 2013). Hazardous wastes are often generated from particular types of small-scale businesses and households. Some of the hazardous waste generating businesses are dry cleaners, auto repair workshops, hospitals, exterminators, and photo processing centres (Alumur and Kara, 2007). Household Hazardous Waste (HHW) is also included in MSW. HHW can be generated from a number of household products including paint, garden pesticides, pharmaceuticals, photographic chemicals, detergents, personal care products, fluorescent tubes, waste oil, heavy metal containing batteries, wood treated with dangerous substances, etc. (Slack et al., 2005). Recyclable materials of MSW are those which can be fully or partly recycled at recycling centres and can be reused after treatment processes. This category of MSW includes paper, plastics, glass and metals. The third category so called here as garbage includes any waste that is neither hazardous nor recyclable and goes directly to residues disposal sites to get disposed there.

In a real-world scenario, sorting the collected wastes into the above-mentioned main categories is performed by intermediate facilities which are often called Transfer Stations or Screening Centres. These facilities play a significant role in economizing the costs of the waste management system. A transfer station is a processing site used for the temporary deposition of wastes by collection vehicles. Prior to being loaded into larger vehicles, the wastes are sorted and balled into the different sorts (EPA, U.S., 2014).

Concurrently taking multiple waste types as an input, while each has distinct processing flows among the MSWM system's facilities (e.g. transfer stations, recycling-, treatment- and disposal centres), forms a complex network of many interrelated components which requires critical decisions including where to establish these facilities and how to route wastes among the facilities. The MSWM network can be even more complex when the types of hazardous wastes and their required treatment technologies are taken into consideration as a real world case. Different types of hazardous wastes require distinctly different treatment processes and technologies such as incineration and chemical treatment. That is, a compatible treatment technology must be selected based on the waste characteristics (Nema and Gupta, 1999).

In addition, an integrated MSWM system (as the one proposed here) requires consideration of different disposing processes and therefore different disposal centres. In practice, disposal centres for hazardous wastes are different from other disposal centres because more strict regulations and controls such as leachate are applying to them (EPA, NSW, 1996).

The existing mathematical models for the Location-Routing Problem (LRP) in waste management have been focused on hazardous wastes only (Nema and Gupta, 1999; List and Mirchandani, 1991; Jacobs and Warmerdam, 1994; Giannikos, 1998; Alumur and Kara, 2007; Zhao and Zhao, 2010 and Samanlioglu, 2013). A comprehensive, integrated model for the optimization of the locations of the waste management system's all components and routes among them is proposed in this paper. This is a novel approach because the existing approaches ignore the locations of recycling centres and do not include transfer stations even though these facilities are inseparable parts of a waste management system. Optimization cannot be achieved without considering their interrelations with the other system's components. Mathematical models can be utilized to design the system by describing objectives, component interactions and possible management strategies. A comprehensive mathematical model can provide systemic means with which the decision-makers can make an optimal management plan (Nema and Gupta, 1999). This paper aims to provide a systemic mathematical model for MSWM to optimize the locations of its components and provide the optimum routing plan for different types of waste transportation flows in order to minimize the total cost of transportation and facility establishment.

2 Municipal Solid Waste Management

Modelling of MSWM has been widely studied in the past several decades. The early models on MSWM dealt with specific aspects of the problem such as vehicle routing or transfer station siting while having practical shortcomings such as neglect of recycling centres and oversimplicity of the model with a single waste type only. The latest researches have focused on refinement of various optimization approaches for the development of more practical models of reliable MSWM (Sharholy et al., 2008). Comprehensive reviews and classifications of the proposed models in waste management are presented in Berger et al. (1991) and Tanskanen (2000).

Among the developed models on different aspects of waste management, LRP has been particularly applied to hazardous waste management. In LRP, decisions should be made simultaneously for the establishment of a single or a set of facilities and the determination of a number of routes for each facility in order to minimize the fixed cost of opening facilities and the cost of shipment between the facilities via the determined routes (Lin et al., 2014).

Development of mathematical models for the optimal site selection of treatment and disposal centres and for the efficient transportation routes from waste sources to these facilities have been addressed in many researches (List and Mirchandani, 1991; Revelle et al.,1991; Jacobs and Warmerdam, 1994; Current and Ratick,1995; Giannikos, 1998; Nema and Gupta,2003; Alumur and Kara ,2007; Zhao and Zhao, 2010; Samanlioglu, 2013 and Boyer et al.,2013). Minimization of the total cost consisting of the opening cost of the facilities and the cost of transportation in the network together with minimization of the risks which are measured by people's exposure to the facilities and routes have been the main objective of their research.

Concurrent optimization of site selection and routing in a waste management system has been studied by some researchers. Zografos and Samara (1990) utilized a goal-programming approach to model a hazardous waste management system. Their main objective is to minimize the total travelling time and risks. However, they considered only a single type of hazardous waste and allocation of only one treatment centre to each generation node. List and Mirchandani (1991) proposed a model for a hazardous waste location-routing problem while considering only treatment and disposal centres. Revelle et al. (1991) developed a mathematical model specifically for nuclear waste to locate storage facilities and select shipment routes while only one type of waste was taken into account. Jacobs and Warmerdam (1994) proposed a mathematical model for a hazardous waste LRP to minimize a linear combination of costs and risks in time while site selection of the storage and disposal facilities were included in their addressed problem.

Current and Ratick (1995) included equity in addition to costs and risks in their developed model. Their model aims to maximize the equity of the risks of the system's components and to minimize other costs and risks. Wyman and Kuby (1995) also presented a model for a hazardous waste management system aiming to optimize the total cost, inequity and the associated risks and to choose the optimum treatment technology. Giannikos (1998) utilized a goal-programming approach to a hazardous waste LRP. In addition to typical cost objective functions, an equitable distribution of disutility caused by the operation of treatment centres was also included in the proposed model.

Different types of hazardous wastes and compatibility between waste types and treatment technologies have been important real-world constraints in some researches. Nema and Gupta (2003) addressed a hazardous waste LRP considering different types of wastes and waste-technology compatibility for treatment. They proposed a model to select the locations of treatment and disposal centres and to route wastes to these facilities without considering recycling centres in the network. Alumur and Kara (2007) developed a mathematical model for a hazardous waste LRP to minimize the total cost of transportation and facilities establishment and to minimize the transportation risks which are measured by the number of people who are exposed to the allocated routes. They studied site selection of disposal and treatment centres and the routing problem of different types of hazardous wastes from generation nodes to compatible treatment centres and from treatment centres to disposal facilities while recycling centres were not considered in their addressed problem. Zhao and Zhao (2010) proposed a goal-programming optimization approach to a hazardous waste management system to select the locations of treatment and disposal centres, and to route multiple hazardous wastes. They also studied the routing problem of hazardous wastes from generation nodes to compatible treatment centres and from treatment centres to disposal facilities.

Recycling centres have been factored in by a few researchers. Samanlioglu (2013) developed a more comprehensive model based on the models introduced by Alumur and Kara (2007) and Zhao and Zhao (2010) by taking some additional real-world aspects into account, such as site selection of recycling centres and determination of waste routes to and from recycling centres. Boyer et al. (2013) proposed a mathematical model for a single type hazardous waste location-routing problem when site selection of recycling centres and direct routing of residues from generation nodes to disposal centres are applied without considering different waste types and waste-technology treatment compatibility constraints.

Regardless of waste types and objective functions, the fundamental principle of our defined problem and proposed model are close to those of Alumur and Kara (2007), Samanlioglu (2013) and Boyer et al. (2013). However, our model covers recycling centres and routing of wastes to and from these centres which were not considered by Alumur and Kara (2007). Moreover, Samanlioglu (2013) and Alumur and Kara (2007) did not study the ability of direct routing between generation nodes and disposal centres in their proposed models. Beyond these differences, different types of waste and waste-technology treatment compatibility are also included in our model unlike similar approach by Boyer et al. (2013). In addition, intermediate transfer stations and distinct disposal facilities for hazardous and non-hazardous waste residues have not been taken into consideration so far in the literature. In summary, a new location-routing problem for an integrated MSWM system is formulated in this paper. Considering real-world aspects, our mathematical model is presented to minimize the total cost of the system including the transportation costs and the opening costs of the system's facilities. The formulation is also tested with real data acquired across New South Wales, Australia.

3 Location-Routing Problem

A schematic display of our defined problem is presented in Figure 1. The diagram in Figure 1 starts with transportation of wastes from generation nodes to transfer stations ($\boldsymbol{x}_{i,j}$) where the wastes are sorted and balled into the recyclable, hazardous and garbage balls. After the sorting process, the balled wastes are sent to their distinct destinations by larger vehicles. Recyclable wastes are transferred to recycling centres ($\boldsymbol{l}_{i,j}$); hazardous wastes are sent to treatment centres with compatible technologies ($\boldsymbol{y}_{w,i,j}$) and garbage which are neither hazardous nor recyclable is transported to non-hazardous disposal centres ($\boldsymbol{p}_{i,j}$). After the treatment process, a waste mass is reduced ($\boldsymbol{r}_{w,q}$) and the parts which are recyclable are sent to recycling centres ($\boldsymbol{k}_{i,j}$)

and non-recyclable waste residues are transported to final hazardous disposal centres ($\boldsymbol{z}_{i,j}$). At the recycling centres, after the recycling process, recyclable wastes are sent to the markets or other factories (β_i) and the generated waste residues are sent to the final non-hazardous disposal centres ($\boldsymbol{v}_{i,j}$).

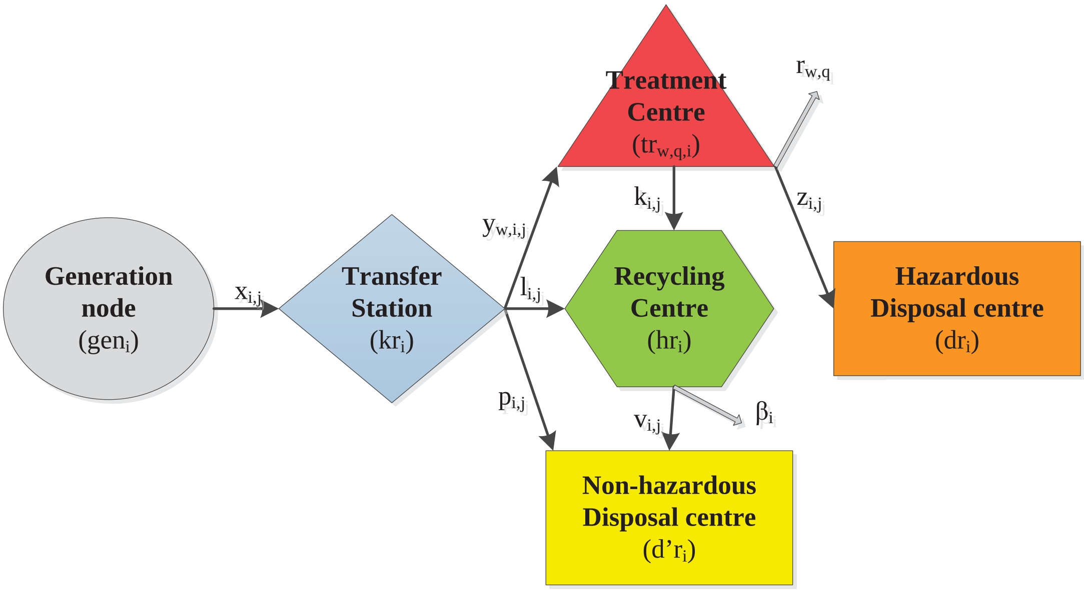

Figure 1. Schematic view of the defined problem

The defined problem consists of concurrent site selection of the locations of the system's facilities (e.g. transfer stations, treatment-, recycling- and disposal centres) from the candidate locations and the determination of routes and amounts of shipments among the selected locations to minimize the total cost of transportation and facility establishment.

4 Mathematical model

A mixed integer programming is proposed in this study to formulate the defined MSW location-routing problem with respect to the minimization of the total cost of transportation and facilities establishment. The nodes of the transportation network consist of generation nodes, potential transfer stations, potential treatment facilities, potential recycling centres, potential hazardous disposal centres, potential non-hazardous disposal centres, and a combination of any of the above. Our mathematical model is formulated to determine: (i) the locations of transfer stations and the routes for wastes to be transported to the transfer stations, (ii) the locations of treatment centres and their adopted technologies, and the routes for different types of hazardous wastes to be transported to compatible treatment centres, (iii) the locations of recycling centres and the routes for recyclable wastes and waste residues to be transported to the recycling centres, (iv) the locations of non-hazardous disposal centres and the routes for waste residues to be transported to these centres, and (v) the locations of hazardous disposal centres and the routes for hazardous waste residues to be transported to these centres. The defined problem can be proven to be NP-hard for large scales (Alumur and Kara, 2007; Samanlioglu, 2013).

The sets, parameters and decision variables of our model are presented below.

Sets:

$N = (V,A)$ is a transportation network of nodes V and arcs A

$G = \{1,\ldots,g\}$ is a set of waste generation nodes, $G \in V$

$K = \{1,\ldots,k\}$ is a set of potential transfer station nodes, $K \in V$

$T = \{1,\ldots,t\}$ is a set of potential treatment nodes, $T \in V$

$D = \{1,\ldots,d\}$ is a set of potential hazardous disposal nodes, $D \in V$

$D' = \{1,\ldots,d'\}$ is a set of potential non-hazardous disposal nodes, $D' \in V$

$H = \{1,\ldots,h\}$ is a set of potential recycling nodes, $H \in V$

$W = \{1,\ldots,w\}$ is a set of hazardous waste types

$Q = \{1,\ldots,q\}$ is a set of treatment technologies

Parameters:

cc_{ij} is a transportation cost per unit of waste on link $(i,j) \in A$, $i \in G$, $j \in K$

c_{ij} is a transportation cost per unit of hazardous waste on link $(i,j) \in A$, $i \in K$, $j \in T$

cz_{ij} is a transportation cost per unit of hazardous waste residue on link $(i,j) \in A$, $i \in T$, $j \in D$

cv_{ij} is a transportation cost per unit of non-hazardous waste residue on link $(i,j) \in A$, $i \in H$, $j \in D'$

cr_{ij} is a transportation cost per unit of recyclable waste on link $(i,j) \in A$, $i \in K$, $j \in H$

cr'_{ij} is a transportation cost per unit of recyclable waste residue on link $(i,j) \in A$, $i \in T$, $j \in H$

cn_{ij} is a transportation cost per unit of garbage waste on link $(i,j) \in A$, $i \in K$, $j \in D'$

fk_i is a fixed cost of opening a transfer station at node $i \in K$

$fc_{q,i}$ is a fixed cost of opening a treatment technology $q \in Q$ at node $i \in T$

fd_i is a fixed cost of opening a hazardous disposal centre at node $i \in D$

fd'_i is a fixed cost of opening a non-hazardous disposal centre at node $i \in D'$

fh_i is a fixed cost of opening a recycling centre at node $i \in H$

gn_i is an amount of waste generated at generation node $i \in G$

$ph_{w,i}$ is a proportion of hazardous waste type $w \in W$ sorted at transfer station node $i \in K$

pr_i is a proportion of recyclable waste sorted at transfer station node $i \in K$

pg_i is a proportion of garbage waste sorted at transfer station node $i \in K$

$r_{w,q}$ is a proportion of mass reduction of hazardous waste type $w \in W$ treated with technology $q \in Q$

$\alpha_{w,q}$ is a proportion of recycling of hazardous waste type $w \in W$ treated with technology $q \in Q$

β_i is a proportion of total waste recycled at node $i \in H$

$tc_{q,i}$ is a capacity of treatment technology $q \in Q$ at node $i \in T$

rc_i is a capacity of recycling centre at node $i \in H$

dc_i is a capacity of hazardous disposal centre at node $i \in D$

$d'c_i$ is a capacity of non-hazardous disposal centre at node $i \in D'$

sc_i is a capacity of transfer station at node $i \in K$

$tc^m_{q,i}$ is the minimum amount of hazardous waste required to establish treatment technology $q \in Q$ at node $i \in T$

rc^m_i is the minimum amount of recyclable waste required to establish a recycling centre at node $i \in H$

dc^m_i is the minimum amount of hazardous waste residue required to establish a hazardous disposal centre at node $i \in D$

$d'c_i^m$ is the minimum amount of garbage and non-hazardous waste residue required to establish a non- hazardous disposal centre at node $i \in D'$

sc_i^m is the minimum amount of waste required to establish a transfer station at node $i \in K$

$yn_{w,q}$ is 1 if hazardous waste type $w \in W$ is compatible with technology $q \in Q$; or 0 otherwise

Decision variables:

$\boldsymbol{x}_{i,j}$ is an amount of waste transported through link $(i,j) \in A$, $i \in G$, $j \in K$

$\boldsymbol{y}_{w,i,j}$ is an amount of hazardous waste type $w \in W$ transported through link $(i,j) \in A$, $i \in K$, $j \in T$

$\boldsymbol{l}_{i,j}$ is an amount of recyclable waste transported through link $(i,j) \in A$, $i \in K$, $j \in H$

$\boldsymbol{p}_{i,j}$ is an amount of garbage waste transported through link $(i,j) \in A$, $i \in K$, $j \in D'$

$\boldsymbol{k}_{i,j}$ is an amount of treated recyclable waste residue transported through link $(i,j) \in A$, $i \in T$, $j \in H$

$\boldsymbol{v}_{i,j}$ is an amount of waste residue transported through link $(i,j) \in A$, $i \in H$, $j \in D'$

$\boldsymbol{z}_{i,j}$ is an amount of hazardous waste residue transported through link $(i,j) \in A$, $i \in T$, $j \in D$

$\boldsymbol{kr}_i$ is an amount of waste transferred at node $i \in K$

$\boldsymbol{tr}_{w,q,i}$ is an amount of hazardous waste type $w \in W$ treated at node $i \in T$ with technology $q \in Q$

$\boldsymbol{dr}_i$ is an amount of hazardous waste residue disposed at node $i \in D$

$\boldsymbol{d'r}_i$ is an amount of non-hazardous waste residue disposed at node $i \in D'$

$\boldsymbol{hr}_i$ is an amount of waste recycled at node $i \in H$

$\boldsymbol{f}_{q,i}$ is 1 if treatment technology $q \in Q$ is established at node $i \in T$; or 0 otherwise

$\boldsymbol{dz}_i$ is 1 if hazardous disposal centre is established at node $i \in D$; or 0 otherwise

$\boldsymbol{d'z}_i$ is 1 if non-hazardous disposal centre is established at node $i \in D'$; or 0 otherwise

$\boldsymbol{b}_i$ is 1 if recycling centre is established at node $i \in H$; or 0 otherwise

$\boldsymbol{a}_i$ is 1 if transfer station is established at node $i \in K$; or 0 otherwise.

The main objective of the problem is to minimize the total cost under the given constraints as follows:

$$Minimize\ f(x) = \sum_{i\in G}\sum_{j\in K} cc_{i,j}\boldsymbol{x}_{i,j} + \sum_{i\in K}\sum_{j\in T}\sum_{w\in W} c_{i,j}\,\boldsymbol{y}_{w,i,j}$$

$$+ \sum_{i\in K}\sum_{j\in H} cr_{i,j}\,\boldsymbol{l}_{i,j} + \sum_{i\in K}\sum_{j\in D'} cn_{i,j}\,\boldsymbol{p}_{i,j} + \sum_{i\in T}\sum_{j\in H} cr'_{i,j}\,\boldsymbol{k}_{i,j} + \sum_{i\in H}\sum_{j\in D'} cv_{i,j}\,\boldsymbol{v}_{i,j}$$

$$+ \sum_{i\in T}\sum_{j\in D} cz_{i,j}\,\boldsymbol{z}_{i,j} + \sum_{i\in K} fk_i\,\boldsymbol{a}_i + \sum_{i\in T}\sum_{q\in Q} fc_{q,i}\boldsymbol{f}_{q,i} + \sum_{i\in H} fh_i\,\boldsymbol{b}_i + \sum_{i\in D} fd_i\,\boldsymbol{dz}_i + \sum_{i\in D} fd'_i\boldsymbol{d'z}_i \qquad (1)$$

subject to

$$gn_i = \sum_{j\in K}\boldsymbol{x}_{i,j} \quad \forall i \in G \qquad (2)$$

$$\sum_{i\in G} x_{i,j} = \boldsymbol{kr}_j \quad \forall j \in K \qquad (3)$$

$$\sum_{w\in W} ph_{w,i}\, \boldsymbol{kr}_i = \sum_{j\in T}\sum_{w\in W} \boldsymbol{y}_{w,i,j} \quad \forall i \in K \tag{4}$$

$$pr_i\, \boldsymbol{kr}_i = \sum_{j\in H} \boldsymbol{l}_{i,j} \qquad \forall i \in K \tag{5}$$

$$(1 - \sum_{w\in W} ph_{w,i} - pr_i)\, \boldsymbol{kr}_i = \sum_{j\in D'} \boldsymbol{p}_{i,j} \qquad \forall i \in K \tag{6}$$

$$\sum_{i\in K} \boldsymbol{y}_{w,i,j} = \sum_{q\in Q} \boldsymbol{tr}_{w,q,j} \qquad \forall w \in W\,, \forall j \in T \tag{7}$$

$$\sum_{w\in W}\sum_{q\in Q} \boldsymbol{tr}_{w,q,i}\,(1 - r_{w,q})(1 - \alpha_{w,q}) = \sum_{j\in D} \boldsymbol{z}_{i,j} \quad \forall i \in T \tag{8}$$

$$\sum_{w\in W}\sum_{q\in Q} \boldsymbol{tr}_{w,q,i}\,(1 - r_{w,q})\,\alpha_{w,q} = \sum_{j\in H} \boldsymbol{k}_{i,j} \quad \forall i \in T \tag{9}$$

$$\sum_{i\in T} \boldsymbol{k}_{i,j} + \sum_{i\in K} \boldsymbol{l}_{i,j} = \boldsymbol{hr}_j \quad \forall j \in H \tag{10}$$

$$\boldsymbol{hr}_i\,(1 - \beta_i) = \sum_{j\in D'} \boldsymbol{v}_{i,j} \quad \forall i \in H \tag{11}$$

$$\sum_{i\in T} \boldsymbol{z}_{i,j} = \boldsymbol{dr}_j \qquad \forall j \in D \tag{12}$$

$$\sum_{i\in H} \boldsymbol{v}_{i,j} + \sum_{i\in K} \boldsymbol{p}_{i,j} = \boldsymbol{d'r}_j \qquad \forall j \in D' \tag{13}$$

$$\boldsymbol{kr}_i \le sc_i\, \boldsymbol{a}_i \qquad \forall i \in K \tag{14}$$

$$\sum_{w\in W} \boldsymbol{tr}_{w,q,i} \le tc_{q,i}\, \boldsymbol{f}_{q,i} \qquad \forall q \in Q\,, \forall i \in T \tag{15}$$

$$\boldsymbol{hr}_i \le rc_i\, \boldsymbol{b}_i \qquad \forall i \in H \tag{16}$$

$$\boldsymbol{dr}_i \le dc_i\, \boldsymbol{dz}_i \qquad \forall i \in D \tag{17}$$

$$\boldsymbol{d'r}_i \le d'c_i\, \boldsymbol{d'z}_i \qquad \forall i \in D' \tag{18}$$

$$\boldsymbol{kr}_i \ge sc_i^m\, \boldsymbol{a}_i \qquad \forall i \in K \tag{19}$$

$$\sum_{w\in W} \boldsymbol{tr}_{w,q,i} \ge tc_{q,i}^m\, \boldsymbol{f}_{q,i} \qquad \forall q \in Q\,, \forall i \in T \tag{20}$$

$$\boldsymbol{hr}_i \ge rc_i^m\, \boldsymbol{b}_i \qquad \forall i \in H \tag{21}$$

$$\boldsymbol{dr}_i \ge dc_i^m\, \boldsymbol{dz}_i \qquad \forall i \in D \tag{22}$$

$$\boldsymbol{d'r}_i \ge d'c_i^m\, \boldsymbol{d'z}_i \quad \forall i \in D' \tag{23}$$

$$\boldsymbol{tr}_{w,q,i} \le tc_{q,i}\, \boldsymbol{yn}_{w,q} \qquad \forall w \in W\,, \forall q \in Q, \forall i \in T \tag{24}$$

$$(\boldsymbol{x}_{i,j}\,, \boldsymbol{kr}_i, \boldsymbol{y}_{w,i,j}, \boldsymbol{l}_{i,j}, \boldsymbol{p}_{i,j}, \boldsymbol{tr}_{w,q,i}, \boldsymbol{k}_{i,j}, \boldsymbol{hr}_i, \boldsymbol{v}_{i,j}, \boldsymbol{d'r}_i, \boldsymbol{z}_{i,j}, \boldsymbol{dr}_i) \in \{\mathbb{R}^{12}\}^+ \tag{25}$$

$$\left(\boldsymbol{f}_{q,i}, \boldsymbol{dz}_i, \boldsymbol{d'z}_i, \boldsymbol{b}_i, \boldsymbol{a}_i\right) \in \{0,1\}^5 \tag{26}$$

The objective function given in Equation (1) minimizes the total cost including the transportation cost of different waste types and waste residues and the fixed cost of opening transfer stations, treatment-, recycling- and disposal centres. The transportation cost is measured by the unit transportation cost times the amount of shipped wastes on a given link.

Equation (2) is the flow balance constraint of the flows from generation nodes to transfer stations. This constraint ensures that all the generated wastes are transported to transfer stations. Equation (3) indicates the total amount of the transported wastes to transfer stations that have to be sorted and balled at these centres. Equations (4)-(6) show the flows of hazardous wastes, recyclables and garbage regarding their proportions from transfer stations to treatment, recycling and non-hazardous disposal centres, respectively. Equation (7) ensures that all hazardous wastes transported to treatment centres have to be treated. Equations (8) and (9) provide the flow from treatment centres to hazardous disposal centres and recycling centres regarding the ratios of recycling and mass reduction associated with different treatment technologies at treatment centres, respectively. Equation (10) presents the flow from transfer stations and treatment centres to recycling centres. Equation (11) provides the flow of generated residues from recycling centres to non-hazardous disposal centres. Equation (12) shows the flow of hazardous waste residues from treatment centres to hazardous disposal centres ensuring that all the transported hazardous residues to these centres have to be disposed at these centres. Equation (13) is the flow of garbage and generated non-hazardous residues from screening and recycling centres to non-hazardous disposal centres and ensures that the total amount of transported residues to these centres has to be disposed at these centres. Equations (14)-(18) determine the capacity limitation for transfer stations, treatment, recycling and disposal centres, respectively. Equations (19)-(23) ensure that minimum amounts of different waste types and waste residues have to exist in order to open the related facilities i.e. transfer stations, treatment, recycling, disposal centres, respectively. Equation (24) presents the compatibility limitation for treatment of different types of hazardous wastes with different treatment technologies. Equations (25) and (26) are utilized for stating non-negative and binary variables, respectively.

5 Experimental results and discussion

Recently, the amount of waste generation in many countries has increased as a result of population growth, technological advances, increase in trade in chemical products and improved health care. Australia has one of the highest rates of waste generation per capita in the world (ABS, 2012). From 1997 to 2012 the rate of waste generation in Australia has sharply increased by 145% compared with the moderate increase rates of 22% and 64% in the population and gross-value-added respectively. Australia's population is estimated to be 35.5 million by the year 2056 which will place increasing pressure on the natural environment and its resources (ABS, 2013).

New South Wales (NSW) is Australia's most populous state with a population of 7.5 million (ABS, 2014). The total amount of domestic wastes generated in NSW was 3.47 million tonnes between 2012 and 2013 with the total recycling ratio of 46.5% (EPA, NSW, 2014). NSW's hazardous waste tracking system also recorded 260,920 tonnes of hazardous wastes generated within the state in 2010-11 (KMH Environmental, 2013). NSW is divided into 12 Statistical Divisions (SDs) where each SD consists of some Statistical Subdivisions (SSs). Sydney SD is the most populous SD in NSW and includes two-thirds of the state's population. In order to cover the whole state in the model analysis and reach more accurate results, all of its 14 SSs are selected so that 25 nodes represent the whole state with the other 11 SDs. All of the 25 waste generation nodes (excluding Inner Sydney due to dense urban environment) are also assumed to be candidate sites for transfer station, treatment, recycling, and disposals centres, concurrently. The study area that has been considered as a case study is illustrated in Figure 2 and the corresponding key information is summarized in Table 1.

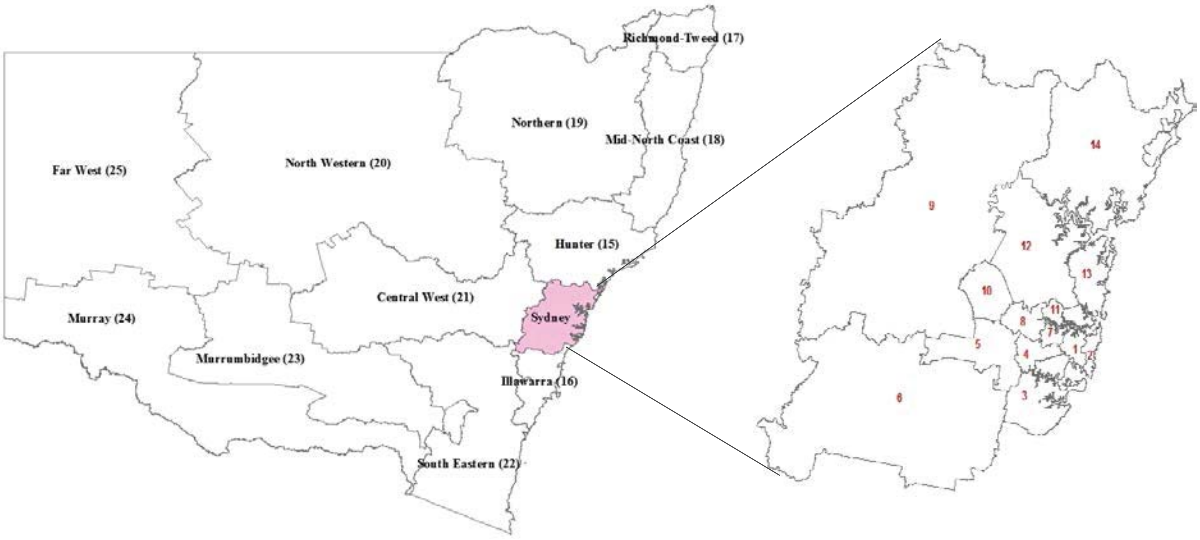

Figure 2. Twenty-five nodes and their corresponding locations on the map of NSW

Node ID	District name	Classification	Population	Waste generation (ton/year)
01	Inner Sydney	SS	334,634	143,236
02	Eastern Suburbs	SS	247,533	109,894
03	St George-Sutherland	SS	445,532	229,229
04	Canterbury-Bankstown	SS	317,412	155,188
05	Fairfield-Liverpool	SS	360,294	173,993
06	Outer South Western Sydney	SS	241,433	125,148
07	Inner Western Sydney	SS	179,892	81,097
08	Central Western Sydney	SS	323,891	155,290
09	Outer Western Sydney	SS	315,985	176,145
10	Blacktown	SS	284,692	155,476
11	Lower Northern Sydney	SS	308,068	142,854
12	Central Northern Sydney	SS	433,118	228,938
13	Northern Beaches	SS	237,514	132,865
14	Gosford-Wyong	SS	306,376	194,638
15	Hunter	SD	624,296	397,438
16	Illawarra	SD	417,901	243,654
17	Richmond-Tweed	SD	232,948	126,013
18	Mid-North Coast	SD	300,006	187,875
19	Northern	SD	180,067	114,002
20	North Western	SD	115,419	59,151
21	Central West	SD	178,840	128,578
22	South Eastern	SD	209,270	120,650
23	Murrumbidgee	SD	154,663	84,297
24	Murray	SD	116,471	67,451
25	Far West	SD	22,817	18,056

Table 1. Features of the case study

To assess our model, the data of recyclables and garbage and their proportions are directly derived from the reported figures from 152 councils within the state (EPA, NSW, 2014). For hazardous wastes, due to the lack of distinct data on the amount of generation in each division, we assume that they are proportional to the populations of the defined nodes. Similar to Alumur and Kara (2007) and Samanlioglu (2013), two types of treatment technologies for hazardous wastes are considered: incineration and chemical treatments. Three types of hazardous wastes are also considered: the first type is compatible with the incineration treatment technology such as clinical wastes; the second type includes wastes which are compatible with chemical treatment e.g. flammable hazardous wastes; and the third type involves wastes which can be treated with both incineration and chemical technologies such as organic hazardous wastes. The proportion of hazardous waste residues which are suitable for recycling after chemical and incineration treatments are also taken as 30% and 0% respectively; and, the percent of waste residues which are sent to disposal centres after the recycling process is assumed to be 5%.

To calculate the total cost of transporting wastes, the amounts of shipment, the transportation distance and the cost of fuel are considered for each pair of the nodes in the network. A code was developed to utilize Google Map API to calculate actual distances derived from centroids of districts polygons using ArcMap 10.2. Here, it is assumed that the average fuel consumption for a truck is 0.3 litre per km, and the average cost of the fuel is $1.5 per litre in Australia. In order to consider special care and equipment associated with transportation of hazardous wastes and hazardous residues, their unit cost of transportation in the network is assumed to be 43% higher than other types of wastes. (Alumur and Kara, 2007; Samanlioglu, 2013). Lastly, other costs of transportation include insurance, driver salary and truck depreciation. The constant factor of 2 is multiplied by the unit cost of transportation in order to include these costs (Boyer et al. 2013).

Based on Alumur and Kara (2007) and Samanlioglu (2013), and considering the current situations of the studied area, the other parameters are determined and listed in Table 2.

Facility \ Parameter	Establishment cost ($M)	Capacity (ton)	Minimum required to establish (ton)
Transfer station	35	1,500	250
Treatment centre	87.5	1,500	50
Recycling centre	35	700	250
Hazardous disposal centre	35	1,500	100
Non-hazardous disposal centre	35	1,500	500

Table 2. Parameters of MSWM facilities

The defined problem has been solved using the General Algebraic Modelling System (GAMS) software with Cplex solver version 12.4.0.1 on a RedHat ® CentOS ® 5.9 Linux server with 8 3.60 GHz Intel ® Xeon ® CPUs with a 198 GB physical memory. Table 3 summarizes the results for the best obtained solution. At this solution, establishment of 7 transfer stations, 1 treatment centre for the each technology, 6 recycling centres, 1 hazardous disposal centre and 1 non-hazardous disposal centre are suggested. Table 4 represents the suggested locations for these facilities. As can be seen, the model results in the minimum possible number of each of the facilities and utilizing the capacity of facilities at the maximum extent. The locations of different types of the facilities are also suggested to be the same in some nodes. Figure 3 illustrates the resulted routing strategy for the defined MSWM system.

Transportation cost ($M)	Total cost ($M)	Total cost lower bound ($M)	CPU time (seconds)	Number of iterations
10.4	815	612	544	2,361,862

Table 3. The optimal solution of the model

Transfer station	Chemical treatment centre	Incineration treatment centre	Recycling centres	Hazardous disposal centres	Non-hazardous disposal centres
2,3,4,8,14,15,21	7	7	2,3,4,14,15,21	7	2,4,15,21

Table 4. Suggested locations for opening facilities at optimal solution

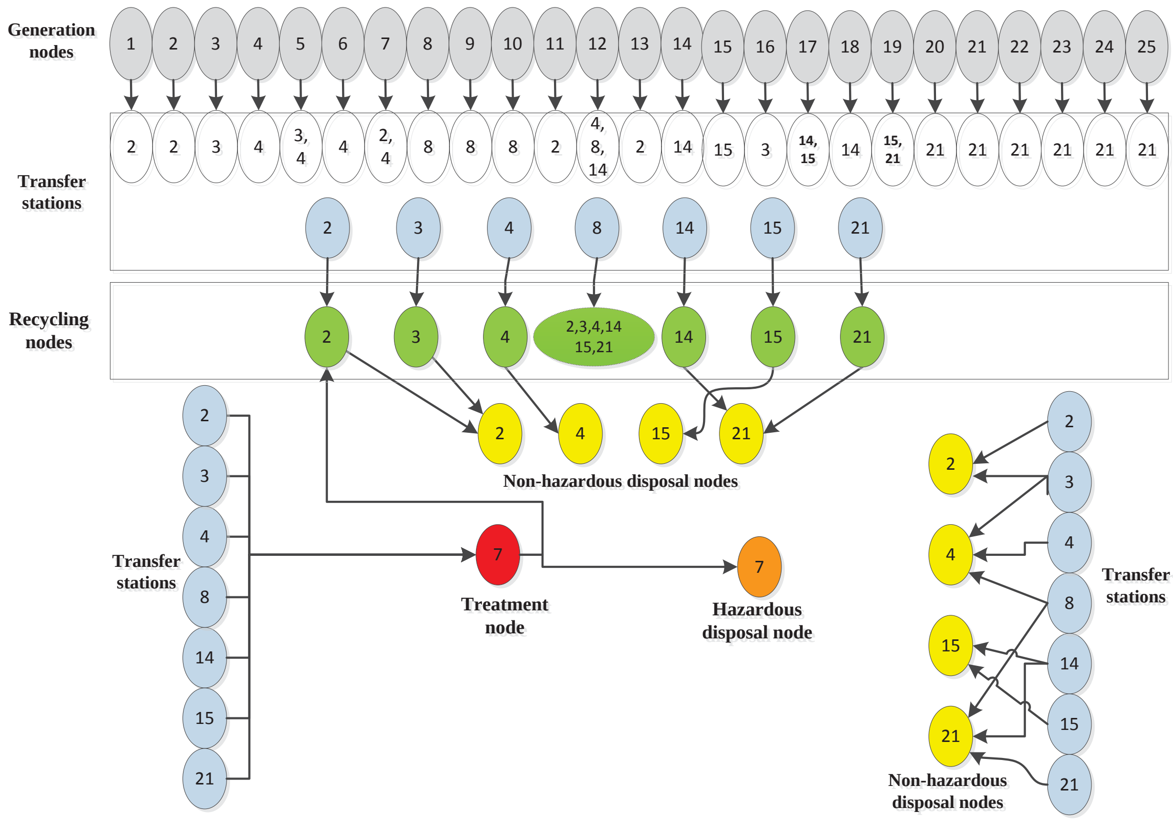

Figure 3. The optimal routing plan

In order to analyse the sensitivity of the cost of transportation and the total cost to the number of transfer stations we varied its optimal value from 7 (Case 1) to 8 (Case 2), 9 (Case 3), 10 (Case 4) and 11 (Case 5). Table 5 summarises the results for the above five cases. The results indicate that, while the total cost rises gradually as the number of transfer stations increases, the transportation cost fluctuates at values near the optimal result and the increase in the number of transfer stations does not reduce the cost of transportation in all cases (Figure 4).

Problem	**Number of transfer stations**	**Transportation cost ($M)**	**Total cost ($M)**
Case 1	7	10.4	815
Case 2	8	16.2	856
Case 3	9	10.4	885
Case 4	10	9.12	919
Case 5	11	13.0	958

Table 5. Transportation and total cost values for different numbers of transfer stations

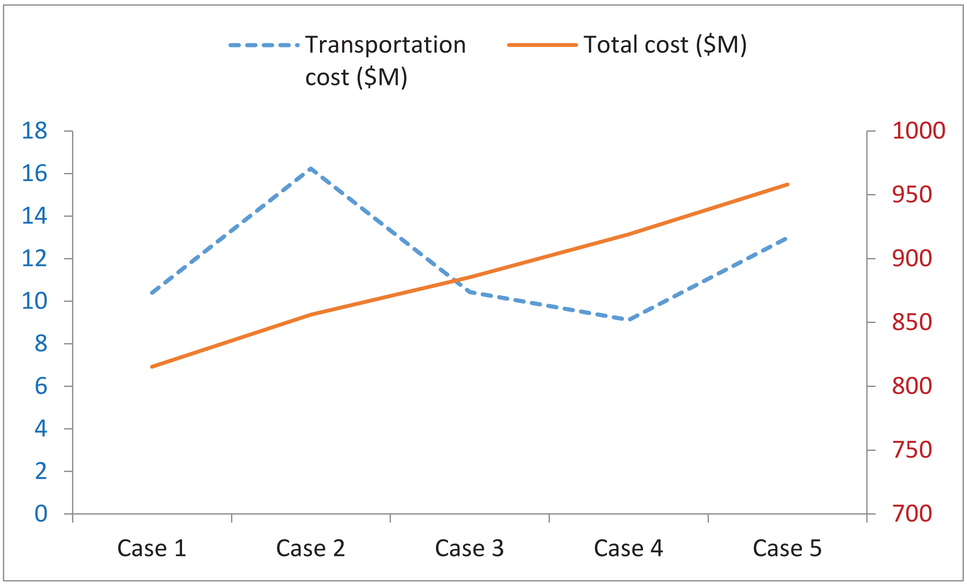

Figure 4. The effect of the number of transfer stations on the system's costs

6 Concluding remarks

In this study, a new location-routing problem for MSWM is proposed by taking applicable aspects of previously developed models in the area of waste management and new real-world features into account. The main contribution of this paper is a mathematical model of MSWM where different waste types are simultaneously factored in, and transfer stations and distinct disposal facilities for hazardous and non-hazardous residues are included in a mixed integer mathematical model. Taking these new aspects into consideration together with the other real-world features such as waste-treatment compatibility, locations of recycling centres, possibility of locating different facilities in the same node, and considering minimum requirement amount constraints, led us to develop a comprehensive and yet more practical model in waste management context. The main objective of the presented study is to find the optimal locations of the MSWM system's facilities consisting of transfer stations, treatment, recycling and disposal centres, and to determine the optimum routing strategy to and from these facilities in order to minimize the total cost of establishment and transportation.

As a case study, the proposed model was tested with the data collected across NSW, Australia. Many real-world features of an integrated MSWM system have been taken into consideration and realistically implemented in the model. The 25 districts based on Statistical Divisions and Statistical Subdivisions zoning approaches were selected to represent the whole state of NSW for the analysis. 24 districts out of the 25 generation nodes were also assumed to be candidate locations for each of the six types of the system's facilities (transfer stations, two types of treatment technologies, recycling centres, hazardous disposal centres and non-hazardous disposal centres) leading to a network with 24×6 potential location nodes. In terms of the number of all the candidate location nodes in the network, the presented application in this study is by far the largest instance studied in the literature.

As it was mentioned by Alumur and Kara (2007) and Samanlioglu (2013) the problem cannot be solved for a large-scale instance even with available high performance computing facilities within a polynomial time. Hence, in order to find a solution for MSWM in a practical time, development of an efficient heuristic method even with a larger size would be worthwhile for future work. A multi-objective optimisation approach to the defined problem can be another future research direction. As the amount of generated wastes is not always deterministic, one can apply stochastic programming techniques to tackle the problem. Finally, factoring in different types of recyclable wastes and waste-recycling technology constraint can improve the applicability of the proposed model in reality.

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